Iteration of critically unbounded meromorphic map

Subhasis Ghora, Tarakanta Nayak

Geometry and Dynamics Laboratory, School of Basic Sciences

Indian Institute of Technology Bhubaneswar

National Science Day 10th Research Scholars' Day 2020 IIT Bhubaneswar

Abstract

Dynamics of $f_\lambda(z) = \lambda + z + tanz$ are investigated for each $\lambda \in \mathbb{C}$. It is proved that

- for each $\lambda \in \mathbb{R}$, the Fatou set, $\mathcal{F}(f_\lambda)$ consist of only two components, one is the upper half plane and another is the lower half plane and both components are completely invariant Baker domains and the Julia set, $\mathcal{J}(f_\lambda)$ is $\mathbb{R} \cup \{\infty\}$.
- It is evinced that for each $\lambda \in \mathbb{C}$, $\Im(\lambda) \geq 0$, $\mathcal{F}(f_\lambda)$ has a completely invariant Baker domain containing the upper half plane.
- It is shown that for each $\lambda \in \mathbb{C}$ with $|2 + \lambda^2| < 1$, there are infinitely many unbounded attracting domains in the lower half plane.

Introduction

Let $f : \to \widehat{\mathbb{C}}$ be a transcendental meromorphic function.

Definition

- *The set of points $z \in \widehat{\mathbb{C}}$ for which the sequence of iterates $\{f^n(z)\}_{n=0}^{\infty}$ is defined is called* the *Fatou set of f.*
- *The Julia set is the complement of the Fatou set of f in $\widehat{\mathbb{C}}$. Details about the Fatou set and the Julia set can be found in [1].*

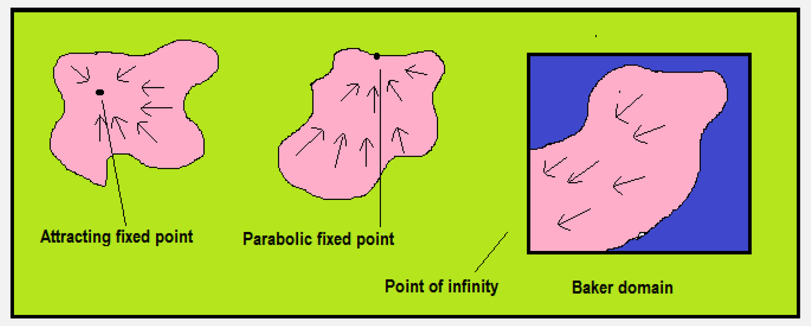
Definition

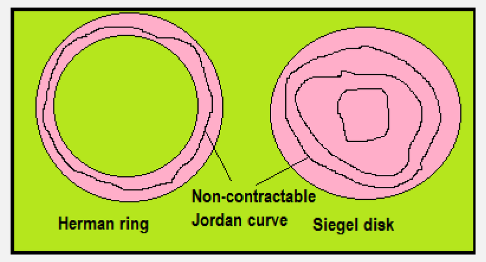
A maximal connected subset of the Fatou set is called a Fatou component.

Periodic Fatou components

A periodic Fatou component can be of 5 types

1. attracting domain
2. parabolic domain
3. Baker domain
4. Siegel disk
5. Herman ring.





A Fatou component U is said to be completely invariant if U is forward invariant as well as backward invariant i.e. $f(U) \subseteq U$ and $f^{-1}(U) \subseteq U$ [2].

Basic properties of f_λ

- The critical points of f_λ are $z_k = \frac{\pi}{2} + \pi k + i\sinh^{-1}1$, when k is odd and $w_k = \frac{\pi}{2} + \pi k - i\sinh^{-1}1$, when k is even.
- For each $\lambda \in \mathbb{C} \setminus \{\pm i\}$ the fixed points of g_λ are all together attracting or repelling or parabolic.
- The functions f_λ and $f_{-\lambda}$ are conformally conjugate.

Results

For $\lambda \in \mathbb{R}$, we have completely described the dynamics of f_λ.

Theorem

For each $\lambda \in \mathbb{R}$, $\mathcal{F}(f_\lambda)$ consist of only two components, one is the upper half plane and another is the lower half plane and both components are completely invariant Baker domains and $\mathcal{J}(f_\lambda) = \mathbb{R} \cup \{\infty\}$.

Existence of a completely invariant Fatou component is evident from the next result.

Theorem

For each $\lambda \in \mathbb{C}$ such that $\Im(\lambda) \geq 0$, $\mathcal{F}(f_\lambda)$ has a completely invariant Baker domain containing the upper half plane.

Let $P = \{\lambda \in \mathbb{C} :$ *all fixed points of f_λ are attracting*$\}$.

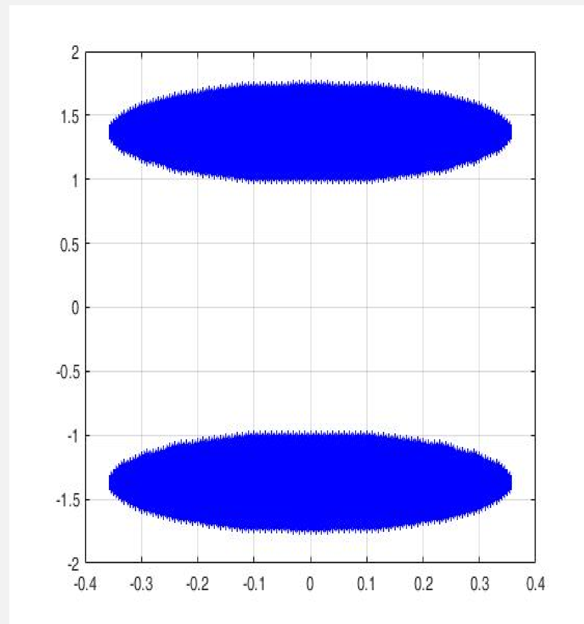


Figure: Image of P

Results

Existence of all possible periodic Fatou components are described whenever the fixed points of f_λ are attracting.

Theorem

For each $\lambda \in \mathbb{C}$ such that $|2 + \lambda^2| < 1$, $\mathcal{F}(f_\lambda)$ has a completely invariant Baker domain containing the upper half plane. Further, there are infinitely many unbounded attracting domains in the lower half plane.

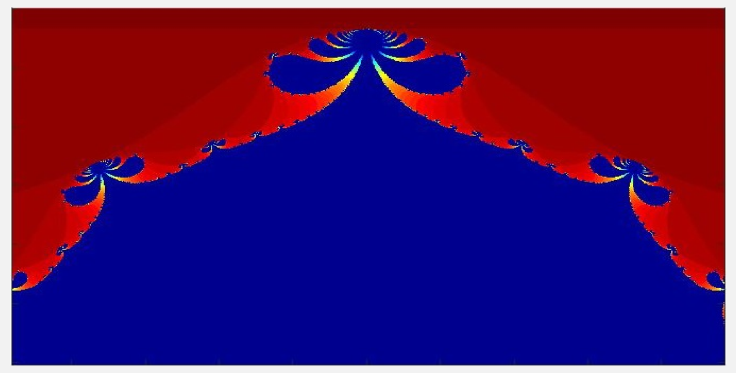


Figure: Blue portion is the unbounded attracting domain and the upper one is the CIFC

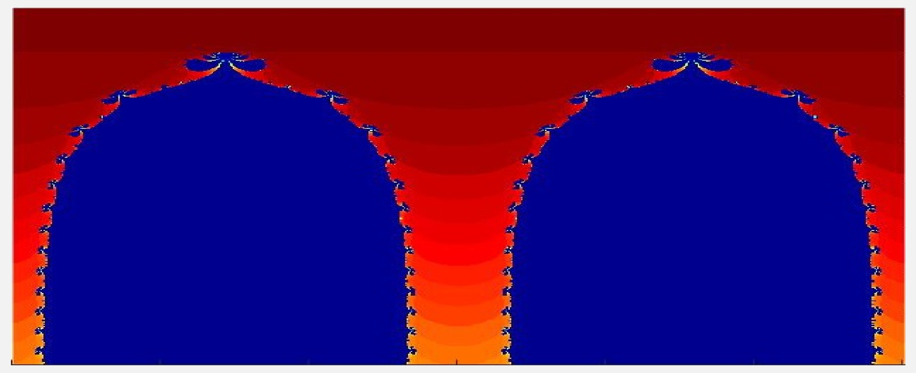


Figure: No. of such attracting domain is infinity

References

[1] Walter Bergweiler.
Iteration of meromorphic functions.
Bulletin of the American Mathematical Society, 29(2):151–189, Jan 1993.

[2] C.L. Cao and Y.F. Wang.
On completely invariant fatou components.
Ark. Mat., 41(2):151–189, 2003.